Modeling a Viral Epidemic With a Concurrent “Misinfodemic”

D. E. Wiggins, A. E. Solomonides PhD

The Kermack-McKendrick Theory

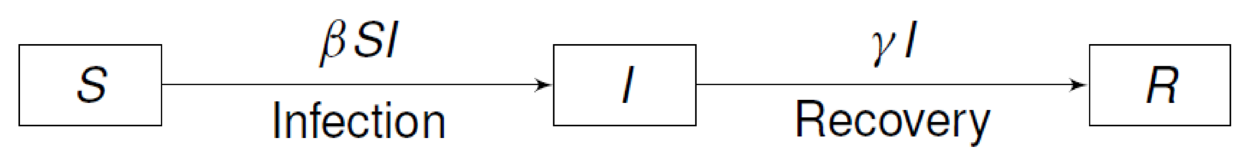


FIGURE: Basic *SIR* transfer diagram

Compartmental ODE Models

Definition (Compartmental Model)

Together, we take (i) a system of time-dependent equations, (ii) its associated transfer diagram, (iii) and the resultant plot over time t to be a compartmental model of infectious disease.

In our case, a compartment refers to one of the population stocks. For instance, in the basic *SIR* model, the susceptible, infected, and recovered populations are the compartments, and the equations model an individuals transfer between compartments.

Remark

In a Kermack-McKendrick population, we assume that the rate of births and rate of deaths are equal, so they are not considered in the transfers.

The Kermack-McKendrick Theory

Definition (SIR Model)

Let S denote the number of susceptible individuals in a population, I the number of infectious individuals, and R the number of recovered individuals. Then, the *SIR* model of infection is the system

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}S = -\underbrace{\beta SI}_{\text{Infection}} \\ \frac{\mathrm{d}}{\mathrm{d}t}I = \underbrace{\beta SI}_{\text{Infection}} - \underbrace{\gamma I}_{\text{Recovery}} \\ \frac{\mathrm{d}}{\mathrm{d}t}R = \underbrace{\gamma I.}_{\text{Recovery}} \end{cases} \tag{1}$$

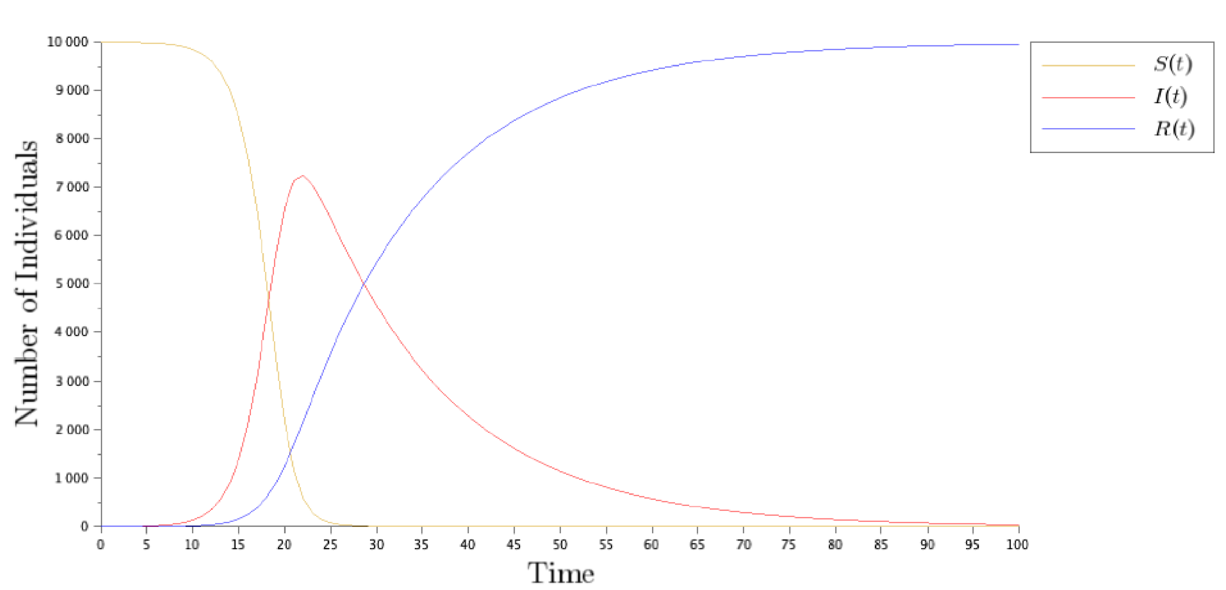


FIGURE: Basic *SIR* plot over time t with $\beta = \frac{7}{10}, \gamma = \frac{1}{15}$

Compartmental ODE Models

Our approach to modeling certain traits of an infectious disease is to gradually build complexity, beginning from the *SIR* model:

i. Identify the characteristic to be added to the altered model.
ii. Construct a logical transfer diagram with transfer ratios.
iii. Use the transfer ratios to write the equations explicitly.
iv. Plot the equations numerically.

Remark

Hereafter, we will write the time derivative $\frac{\mathrm{d}}{\mathrm{d}t}F$ *as* $\dot{F}$.

Information Dispersal

We begin by allowing information spread, where a misinformed population M “learns” via contact with an knowledgeable population K, stratifying the susceptible population into 2 groups:

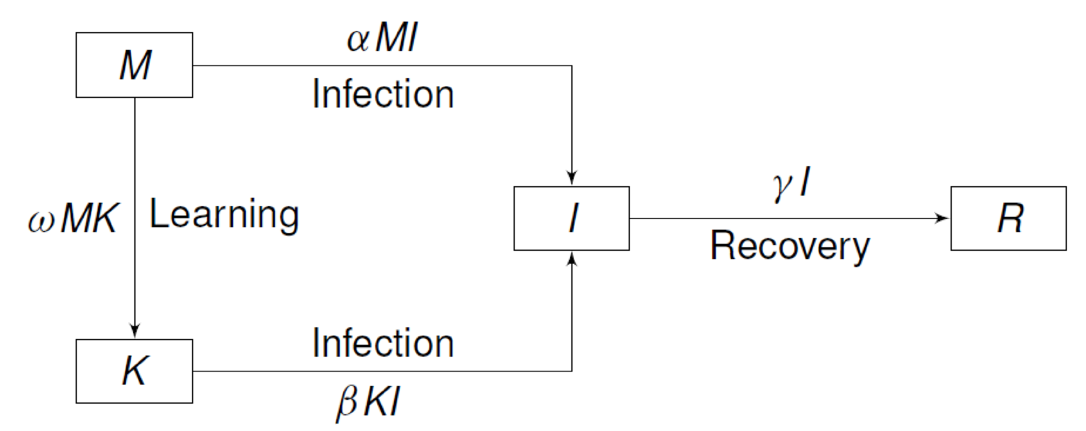


FIGURE: *MKIR* transfer diagram with information dispersal

Converting this into ODEs yields the system

$$\begin{cases} \dot{M} = -\underbrace{\omega MK}_{\text{Learning}} - \underbrace{\alpha MI}_{\text{Infection}} \\ \dot{K} = \underbrace{\omega MK}_{\text{Learning}} - \underbrace{\beta KI}_{\text{Infection}} \\ \dot{I} = \underbrace{(\alpha M + \beta K)I}_{\text{Infection}} - \underbrace{\gamma I}_{\text{Recovery}} \\ \dot{R} = \underbrace{\gamma I.}_{\text{Recovery}} \end{cases} \tag{2}$$

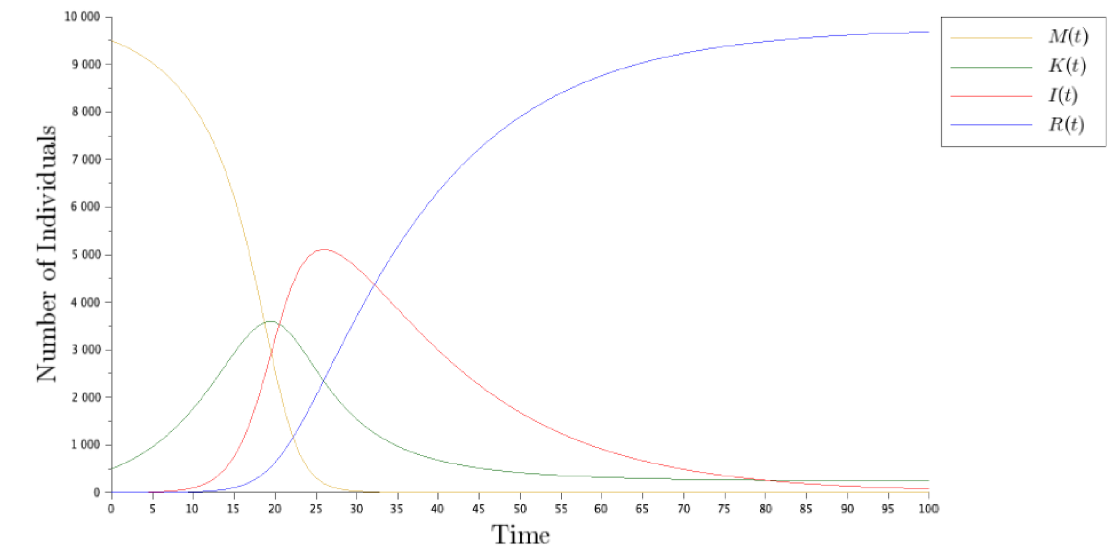


FIGURE: *MKIR* with $\omega = \frac{15}{100}, \alpha = \frac{7}{10}, \beta = \frac{2}{10}, \gamma = \frac{1}{15}$

Exposure Latency

Now, we add an “exposure latency,” where there is a non-contagious exposed period E in between when an individual is susceptible and infected:

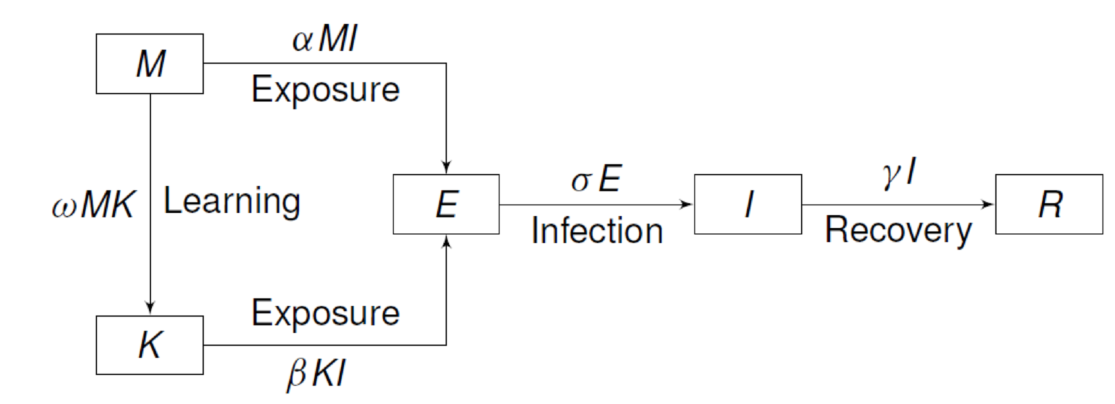


FIGURE: *MKEIR* transfer diagram with exposure latency

$$\begin{cases} \dot{M} = -\underbrace{\omega MK}_{\text{Learning}} - \underbrace{\alpha MI}_{\text{Exposure}} \\ \dot{K} = \underbrace{\omega MK}_{\text{Learning}} - \underbrace{\beta KI}_{\text{Exposure}} \\ \dot{E} = \underbrace{(\alpha M + \beta K)I}_{\text{Exposure}} - \underbrace{\sigma E}_{\text{Infection}} \\ \dot{I} = \underbrace{\sigma E}_{\text{Infection}} - \underbrace{\gamma I}_{\text{Recovery}} \\ \dot{R} = \underbrace{\gamma I.}_{\text{Recovery}} \end{cases} \tag{3}$$

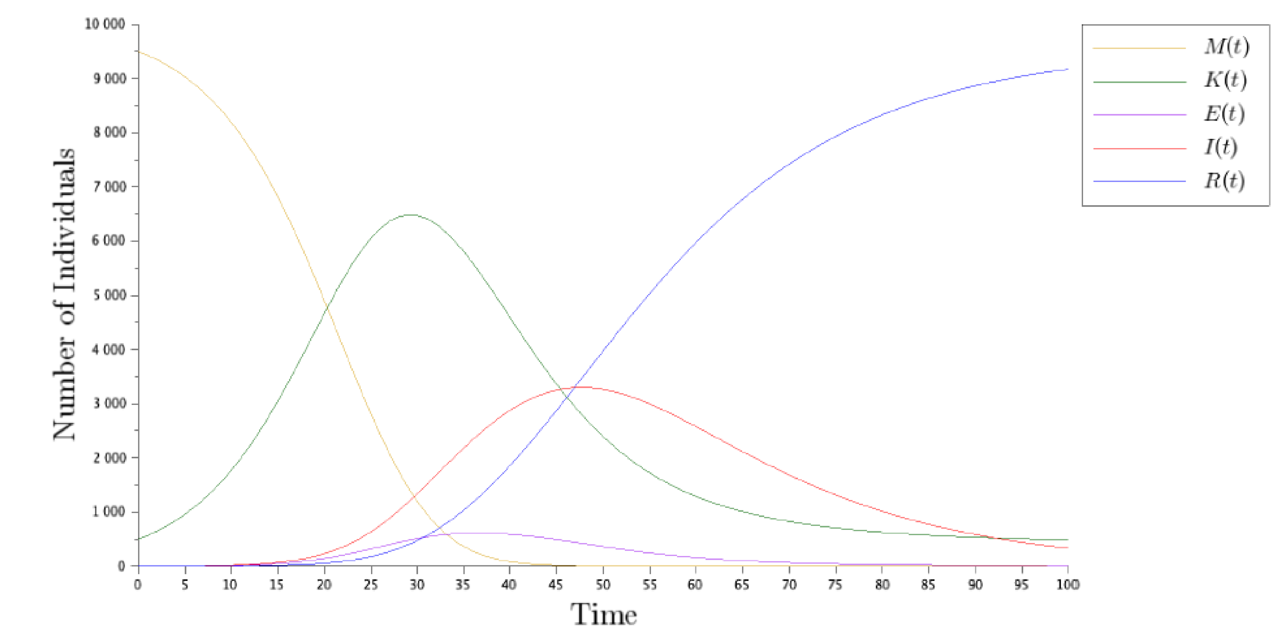
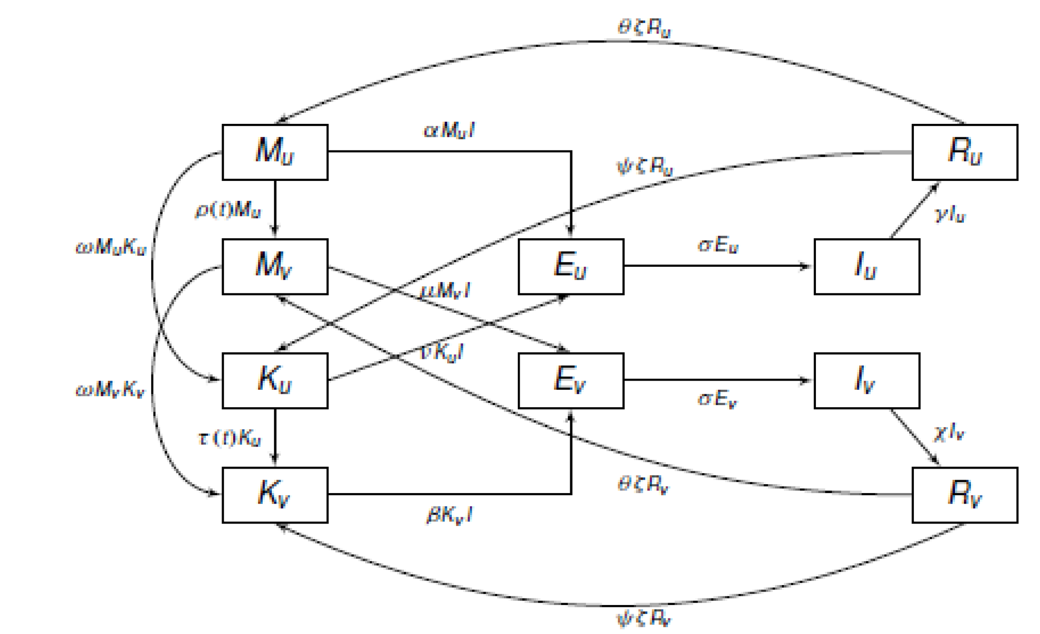


FIGURE: *MKEIR* with $\omega = \frac{15}{100}, \alpha = \frac{7}{10}, \beta = \frac{2}{10}, \sigma = \frac{1}{2}, \gamma = \frac{1}{15}$

Vaccination and Temporary Immunity

Q. What if we were to add a vaccination mechanism and make post-infection immunity temporary?

A. The model becomes ***very*** messy!



$$\begin{cases} \dot{M}_u = -\underbrace{\omega M_u K_u}_{\text{Learning}} - \underbrace{\rho(t) M_u}_{\text{Vaccination}} - \underbrace{\alpha M_u I}_{\text{Exposure}} + \underbrace{\theta \zeta R_u}_{\text{Loss of Immunity}} \\ \dot{M}_v = -\underbrace{\omega M_v K_v}_{\text{Learning}} + \underbrace{\rho(t) M_u}_{\text{Vaccination}} - \underbrace{\mu M_v I}_{\text{Exposure}} + \underbrace{\theta \zeta R_v}_{\text{Loss of Immunity}} \\ \dot{K}_u = \underbrace{\omega M_u K_u}_{\text{Learning}} - \underbrace{\tau(t) K_u}_{\text{Vaccination}} - \underbrace{\nu K_u I}_{\text{Exposure}} + \underbrace{\psi \zeta R_u}_{\text{Loss of Immunity}} \\ \dot{K}_v = \underbrace{\omega M_v K_v}_{\text{Learning}} + \underbrace{\tau(t) K_u}_{\text{Vaccination}} - \underbrace{\beta K_v I}_{\text{Exposure}} + \underbrace{\psi \zeta R_v}_{\text{Loss of Immunity}} \\ \dot{E}_u = \underbrace{(\alpha M_u + \nu K_u) I}_{\text{Exposure}} - \underbrace{\sigma E_u}_{\text{Infection}} \end{cases} \tag{4}$$

$$\begin{cases} \dot{E}_v = \underbrace{(\mu M_v + \beta K_v) I}_{\text{Exposure}} - \underbrace{\sigma E_v}_{\text{Infection}} \\ \dot{I}_u = \underbrace{\sigma E_u}_{\text{Infection}} - \underbrace{\gamma I_u}_{\text{Recovery}} \\ \dot{I}_v = \underbrace{\sigma E_v}_{\text{Infection}} - \underbrace{\chi I_v}_{\text{Recovery}} \\ \dot{R}_u = \underbrace{\gamma I_u}_{\text{Recovery}} - \underbrace{\zeta R_u}_{\text{Loss of Immunity}} \\ \dot{R}_v = \underbrace{\chi I_v}_{\text{Recovery}} - \underbrace{\zeta R_v}_{\text{Loss of Immunity}} \\ I = \underbrace{I_u + I_v,}_{\text{Total Infected}} \end{cases} \tag{5}$$

$\rho(t)$ and $\tau(t)$ are linear vaccination rates “turned on” at time $t = t_v$.

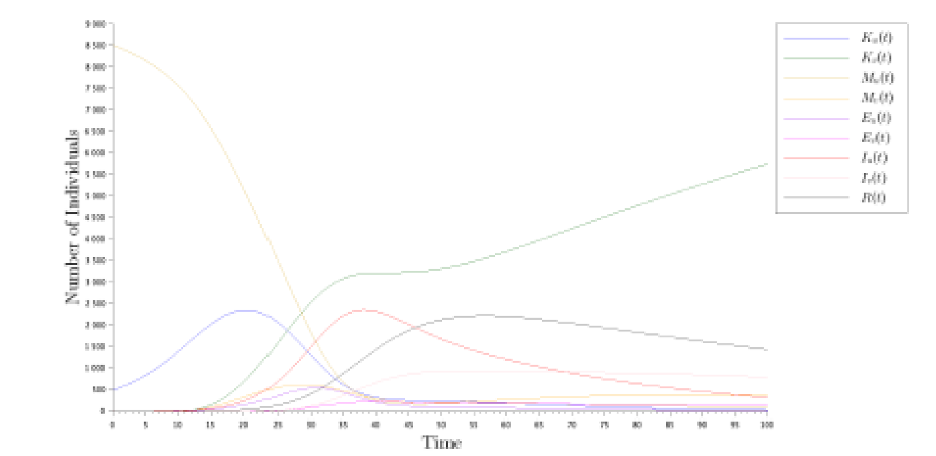


FIGURE: *MKEIVR* plotted over time t, where $R(t) = R_u(t) + R_v(t)$

Dheeran Wiggins joined NorthShore in 2022 as an intern from the Illinois Math and Science Academy under the school’s program for student research. He is now a Math and Physics major at the University of Illinois, Urbana-Champaign. He returned to the Research Institute in 2023 and will be back again this summer to bring this work to completion.

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