PROBLEM SET 08: GENERAL QFT AND DIHEDRAL GROUPS

Exercise 0.1 (Recalling Defintions)**.** Define the

(i) general quantum Fourier transform.
(ii) dihedral group.

Exercise 0.2. Show that the general quantum Fourier transform $\Gamma : \mathbb{C}^n \to \mathbb{C}^n$ is unitary, and thus, is physically realizable in the sense of our evolution axiom.

Exercise 0.3. Derive, using geometric intuition, an underlying set of the dihedral group D_7.[1] Further, evidence that these elements exhibit the usual relations of the dihedral group, where the operation is composition of the rigid motions.[2]

Exercise 0.4. Show that $C_n = \langle r \rangle$ is normal in D_n.

Exercise 0.5. Let N be normal in a group G. Show that for any subgroup $H \leq G$, we have that $H' = (H \cap N)$ is normal in H. Thus, for all $H \leq D_n$, the subgroup $H \cap C_n$ in H is normal.

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[1]Recall that this is the group of rigid motions of a regular heptagon H.

[2]You can either describe the operation informally, by "concatenating" rotations and reflections, or by discussing linear transformations $\mathbb{R}^2 \to \mathbb{R}^2$ restricted to the heptagon $H \hookrightarrow \mathbb{R}^2$ embedded in the plane.