PROBLEM SET 06: REPRESENTATIONS AND THE ABELIAN QFT

Exercise 0.1 (Recalling Defintions)**.** Define

(i) a representation.
(ii) an irreducible representation.
(iii) a character.
(iv) the abelian QFT.

Exercise 0.2. If $\chi(G)$ is the group of all $\chi_g : G \to \mathbb{C}^\times$ defined as in lecture, show that $\chi(G)$ is isomorphic to G itself.

Exercise 0.3. Show that all irreducible representations $G \to \mathrm{GL}(\mathcal{V})$ have $\dim(\mathcal{V}) = 1$.

Exercise 0.4. Assume the fact that $|G/H| = |H^\perp|$, where G/H is the set of H-cosets in G.[1] Show that $H = (H^\perp)^\perp$.

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Date: August 14, 2025.

[1]This is the same notation that we use for a quotient group, since if N is normal, then the quotient G/N is precisely the set of N-cosets in G, using the usual quotient operation.