MATH 405: TEACHER'S COURSE, HONORS PROJECT
ALGEBRA: PREPARATION AND ABSTRACTION

DHEERAN E. WIGGINS

As abstract as the subject of algebra is now, the study of objects like groups, rings, modules, and fields stems from rather concrete questions concerning counting and symmetries. At the middle school and secondary level, algebra is presented via a sequence of rules to solve different classes of equations. Ideally, this approach should prepare students for subjects in college, and beyond, where they will have to recognize that a problem requires the algebra they know, and apply it. While the rules are often presented with minimal defense for their validity, an undergraduate mathematics student soon learns at least one explanation for many them, arising from the properties of abstract algebraic structures. In this honors project, we will consider three standard topics in algebra: linear systems and vector spaces, roots of polynomials and field extensions, and the unit circle and group actions. Namely, we will look at the topics from pre-abstract and abstract perspectives.

How will we approach this material? There will be guided worksheets (labeled HPn in the schedule below) that lead you through the three topics from both perspectives. These will be due by email to Dheeran, and will be assessed on "good faith" attempts. We will also have semi-regular meetings (to be scheduled via email, based on availability) in which you can ask questions and discuss the material with your peers.

You will also be responsible for preparing a final talk (a writeup and an in-person presentation) assessing or critiquing the presentation of one of these three topics to students at secondary and early-college levels. In particular, you will use find specific syllabi from a high school and our university, placing them under scrutiny in their ability to prepare students. Via this case study, you should simultaneously assess whether the abstract perspective could be utilized to illuminate, rather than obscure, the material.

Week	Topic	Task
0	Algebraic Structures and Actions	Attend Lecture
1	Linear Systems via Linear Spaces	Do HP01
2	Polynomials and Adjoining Roots	Do HP02
3	Ways to Rotate a Circle	Do HP03
4	Wrap Up	Begin Writeup
5	Wrap Up	Begin Slides
6	Wrap Up	Submit Writeup
7	Wrap Up	Give Talk

University of Illinois Urbana-Champaign, Illinois, 61801
Email address: dheeran2@illinois.edu

Date: Fall, 2026.