MATH 231: CALCULUS II, HONORS PROJECT
THREE SOLUTIONS TO THE BASEL PROBLEM

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The Basel problem, originally considered in the seventeenth century by Mengoli, and solved nearly a century later by Euler, is a question about the value of a particular series. Mengoli asks us for the value of

$$B = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Since Euler, who gave multiple proofs himself, many solutions to the Basel problem have been found, leveraging techniques from all over mathematics. In this honors project, we will take a look at three solutions:

(i) Euler's original approach, using what we now call the *Weierstrass factorization* of sine.
(ii) an approach via Fourier series and *Parseval's identity* for Fourier coefficients.
(iii) an approach using *Leibniz's rule*[1] of differentiation under the integral sign.

How will we approach this material? There will be guided worksheets (labeled HPn in the schedule below), designed to be worked through like your usual discussion worksheets, that lead you through both the prerequisites and the proofs for solutions (i)–(iii) of the Basel problem. These will be due by email to Dheeran, and will be assessed on "good faith" attempts. We will also have semi-regular meetings (to be scheduled via email, based on availability) in which you can ask questions and discuss the material with your peers.

You will also be responsible for preparing a final talk (slides, and perhaps a recorded/in-person presentation) describing the Basel problem, one solution to it from the list above, and any prerequisites to understand that given solution. This should be done at a level where your Calculus II peers, who have likely not seen the Basel problem before, could reasonably follow along and understand.

Week	Topic	Task (Worksheet or Presentation Preparation)
1	Introduction to the Basel Problem	Do HP01
1	Introduction to the Basel Problem	Make INTRODUCTION Slides
2	Differentiation Under the Integral	Do HP02
2	Differentiation Under the Integral	Make INTRODUCTION Slides
3	Fourier and Parseval	Do HP03
3	Fourier and Parseval	$*$ Slides
4	Euler's Solution	Do HP04
4	Euler's Solution	$*$ Slides
5	Wrap Up	Finish Slides

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[1] also known as *Feynman's trick*