

**MATH 231: CALCULUS II, HONORS PROJECT or  
INTRODUCTION TO THE BASEL PROBLEM**

**Summary.** Our goal is to compute the value of the *Basel series*

$$B = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

But remember, this series is really the limit

$$B = \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{n^2},$$

so for this to be a well-founded question, we should first be sure that  $B$  converges. In this first worksheet, you will study the convergence of  $B$  by using the many series convergence tests we have covered.

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**Exercise 1** (Divergence Test). Use the divergence test on the Basel series  $B$ . Does this tell us anything about its convergence?

*Solution.*

□

**Exercise 2** (Integral Test). Use the integral test to show that  $B$  converges.

*Solution.*

□

*Remark 1* ( $p$ -Series Test). Remember, we actually have a distinct convergence test, proven via the integral test, for series that look like  $B$ . Given a series of the form

$$S = \sum_{n=1}^{\infty} \frac{1}{n^p},$$

the  $p$ -series test tells us that  $S$  converges when  $p > 1$  and diverges when  $p \leq 1$ .

**Exercise 3** (Approximation). Compute the five thousandth partial sum

$$B_{5000} = \sum_{n=1}^{5000} \frac{1}{n^2}.$$

(Obviously, do not do this by hand. For instance, use a tool like WolframAlpha.) Round to the thousandths place.

*Solution.*

□