MATH 231: CALCULUS II, HONORS PROJECT 01
INTRODUCTION TO THE BASEL PROBLEM

Summary. Our goal is to compute the value of the *Basel series*

$$B = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

But remember, this series is really the limit

$$B = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{1}{n^2},$$

so for this to be a well-founded question, we should first be sure that B converges. In this first worksheet, you will study the convergence of B by using the many series convergence tests we have covered.

Exercise 0.1 (Divergence Test)**.** *Use the divergence test on the Basel series B. Does this tell us anything about its convergence?*

Solution.

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Exercise 0.2 (Integral Test)**.** *Use the integral test to show that B converges.*

Solution.

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Remark 0.3 (*p*-Series Test). Remember, we actually have a distinct convergence test, proven via the integral test, for series that look like B. Given a series of the form

$$S = \sum_{n=1}^{\infty} \frac{1}{n^p},$$

the *p*-series test tells us that S converges when $p > 1$ and diverges when $p \leq 1$.

Exercise 0.4 (Approximation)**.** *Compute the five thousandth partial sum*

$$B_{5000} = \sum_{n=1}^{5000} \frac{1}{n^2}.$$

(Obviously, do not do this by hand. For instance, use a tool like WolframAlpha.) Round to the thousandths place.

Solution.

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